Design Engineering – DE1.3 Electronics 1

Solutions to Problem Sheet 3 (Topics 7 - 8)

Note: In many of the solutions below I have written the voltage at node X as the variable X instead of V_X in order to save writing so many subscripts.

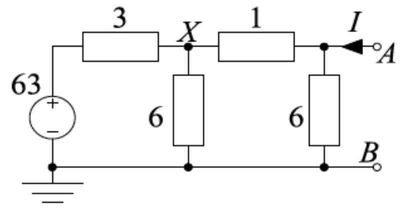
1. (a) The Thévenin equivalent voltage equals the open circuit voltage is 4 V (from potential divider). To obtain the Thévenin resistance, we set the voltage source to 0 (making it a short circuit) and find the resistance of the network to be 1||4 = 0.8Ω.

   (b) The open-circuit voltage is -8V (since the 2 A current flows anticlockwise). To obtain the Thévenin resistance, we set the current source to zero (zero current implies an open circuit), so the resultant network has a resistance of 4 Ω.

2. KCL at node A gives $\frac{A-5}{1}+\frac{A}{4}-I=0$ from which $5A-20-4I=0$ which we can rearrange to give $A=4+0.8I=V_{Th}+R_{Th}I$. We can also rearrange to give $I=-5+\frac{1}{0.8}A=-I_{Nor}+\frac{1}{R_{Nor}}A$.

3. (a)
[Method 1 - circuit manipulation] To calculate the Thévenin equivalent, we want to determine the open-circuit voltage and the Thévenin resistance. To determine the open-circuit voltage, we assume that $I=0$ and calculate V_{AB}. Since $I=0$, we can combine the $1\,\Omega$ and $6\,\Omega$ resistors to give $7\,\Omega$ and then combine this with the $6\,\Omega$ resistor in parallel to give $\frac{42}{13}\,\Omega$. We now have a potential divider so the voltage at point X is $63\times\frac{42/13}{3+42/13}=\frac{98}{3}$. This is then divided by the $1\,\Omega$ and $6\,\Omega$ resistors to give an open-circuit voltage of $\frac{98}{3}\times\frac{6}{7}=28\,\text{V}$. The Thévenin resistance can be found by short-circuiting the voltage source to give $3\,\Omega$ in parallel with $6\,\Omega$ which equals $2\,\Omega$. This is then in series with $1\,\Omega$(to give $3\,\Omega$) and finally in parallel with $6\,\Omega$ to give $2\,\Omega$.

(b)
[Method 2 - Nodal Analysis]. We can do KCL at node X (see diagram below) to get $\frac{X-63}{3}+\frac{X}{6}+\frac{X-A}{1}=0$ which simplifies to $9X-6A=126$ or $3X-2A=42$. We now do KCL at A but include an additional input current I as shown in the diagram. This gives $\frac{A-X}{1}+\frac{A}{6}-I=0$ from which $7A-6X=6I$. Substituting for $6X=4A+84$ gives $3A=84+6I$ or $A=28+2I$. This gives the Thévenin voltage as 28 and the Thévenin/Norton resistance as $2\,\Omega$.Hence the Norton current is $14\,\text{A}$.



4.
(a) KCL @ X gives $\frac{X-14}{1}+\frac{X}{4}+\frac{X}{2}=0$ from which $7X=56\Rightarrow X=8\Rightarrow I=\frac{X}{2}=4\,\text{mA}$.

(b)Finding the Thévenin equivalent of the left three components: we consider the two resistors as a potential divider to give $V_{Th}=14\times\frac{4}{5}=11.2\,\text{V}$. Setting the source to zero (short circuit) gives $R_{Th}=1||4=800\,\Omega$. Hence $I=\frac{V_{Th}}{R_{Th}+2000}=\frac{11.2}{2.8}=4\,\text{mA}$.

5.
From question 4, the left three components have a Thévenin equivalent: $V_{Th}=11.2\,\text{V}$ and $R_{Th}=800\,\Omega$. It follows that the maximum power will be dissipated in R when $R=R_{Th}=800\,\Omega$ (see notes page 5-8). Since the voltage across R will then be $\frac{1}{2}V_{Th}$ the power dissipation will be $\frac{1}{4R_{Th}}V_{Th}^2=39.2\,\text{mW}$.

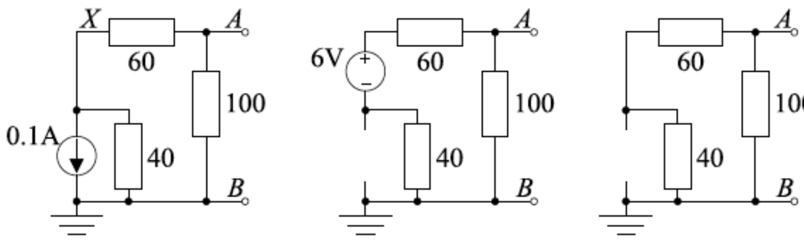
6.

Setting the voltage source to zero gives us the first diagram. Combining $40||(60+100) = 32$ so $X = -0.1 \times 32 = -3.2\,\text{V}$. it follows (potential divider) that $A = -3.2 \times \frac{100}{160} = -2\,\text{V}$.

Now setting the current source to zero gives the second diagram and we have a potential divider giving $V_{AB} = 6 \times \frac{100}{100+60+40} = 3\,\text{V}$.

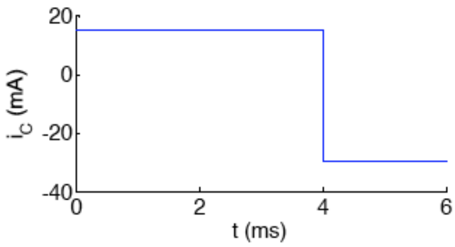
Superposition now gives us $V_{AB} = V_{Th} = -2 + 3 = 1\,\text{V}$.

To find R_{Th} we set both sources to zero and find the resultant resistance of $100||(60+40) = 100||100 = 50\,\Omega$.



7.

$i = C\frac{dv}{dt}$. $\frac{dv}{dt}$ is 3000 V/s for the first 4 ms and −6000 V/s for the next 2 ms. So $i = +15$ or -30 mA.



8.

When x changes from low to high, y will change from high to low. The maximum current is 2 mA so $\frac{dy}{dt} = -\frac{i}{C} = -50\,\text{MV/s}$. So the time to fall from 5 V to 1.5 V is $\frac{3.5}{50} \times 10^{-6} = 70\,\text{ns}$.